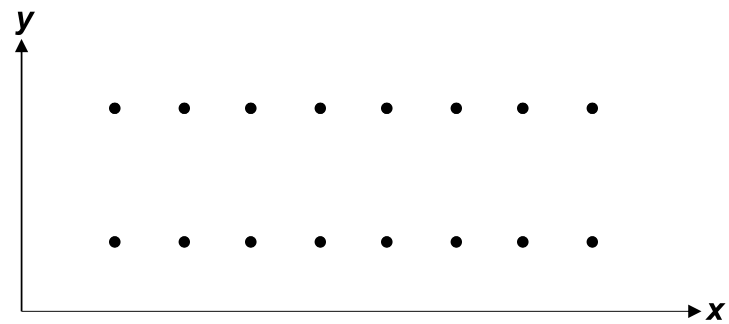
Spatial Statistics



Classical Stats

Obs.	Value
1	23
2	18
3	12
i	Y_i

Spatial Stats

Obs.	Value	*x*	*y*
1	23	1	1
2	18	1	4
3	12	2	1
i	Y_i	x_i	y_j

Spatial Statistics

Assumptions

Assumption	Classical	Spatial
$E(Y_i)$	μ_y	μ
$Var(Y_i)$	σ^2_y	
$Var(Y_{i+h} - Y_i)$		$2\gamma(h)$

Where h = the distance between Y_i and Y_{i+h}

Spatial Statistics
Semivariance

Pairwise:

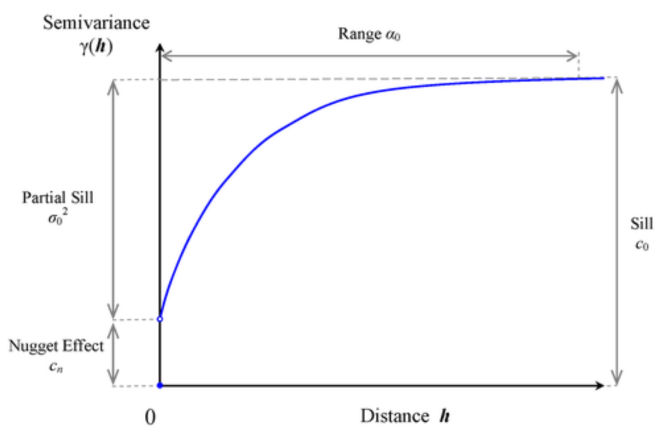
$$\gamma(h) = 1/2(y_{i+h} - y_i)^2$$

Average:

$$\gamma(h) = (1/2n)\sum_{i=1}^{n}(y_{i+h} - y_i)^2$$

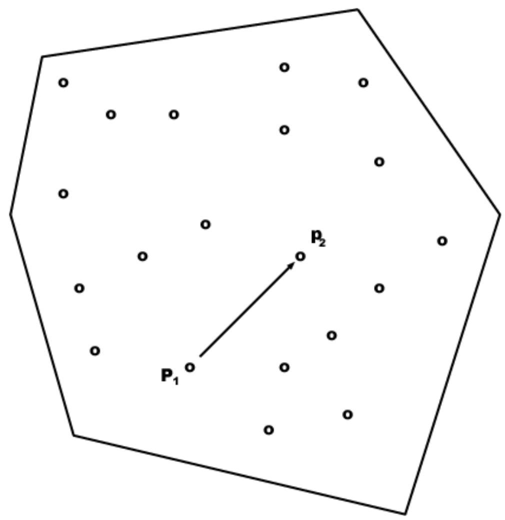
Where:
- $\gamma(h)$ = semivariance at distance h
- n = the number of pairs h units apart
- y_i = the value of y at point i

Spatial Statistics
Semivariogram

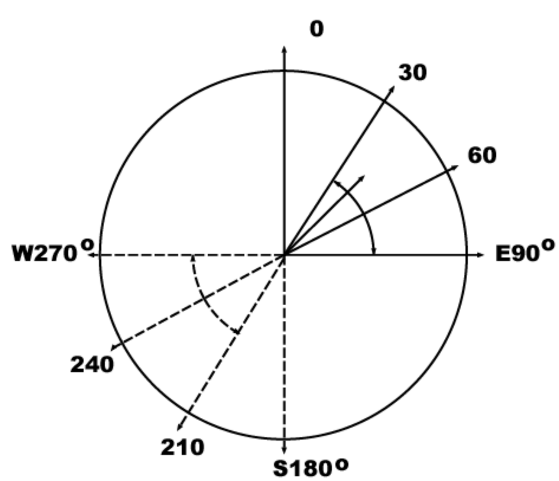


Source: SAS Variogram Procedure Documentation.

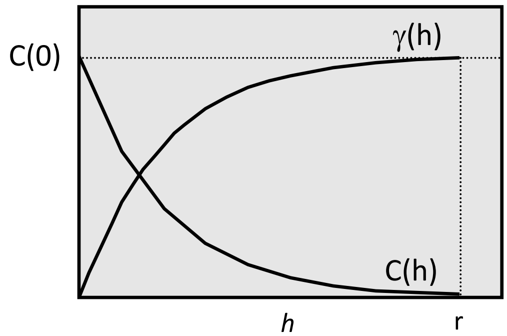
Spatial Statistics
Semivariance Calculation



Spatial Statistics
Isotropic and Anisotropic Variation



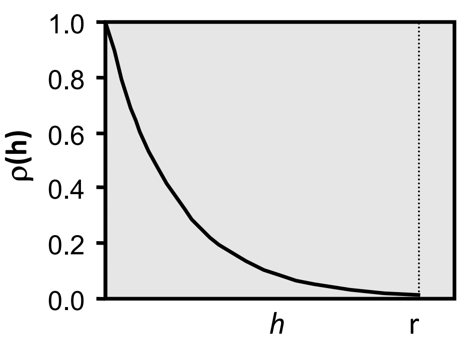
Spatial Statistics
Relationship to Covariance



$$\gamma(h) = C(0) - C(h)$$

$$C(h) = C(0) - \gamma(h)$$

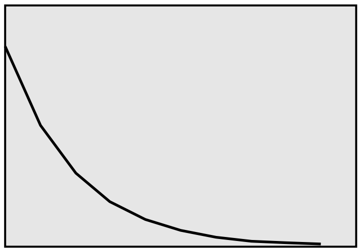
Spatial Statistics
Relationship to Correlation



$$\rho(h) = C(h) / C(0)$$

$$\gamma(h) = C(0)[1 - \rho(h)]$$

Spatial Statistics
Spatial Covariance Models

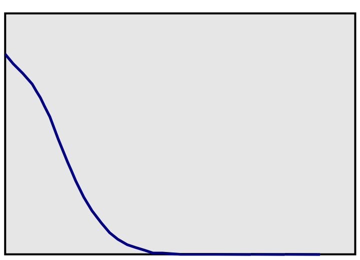


Exponential Model:

$C(h) = \sigma^2 e^{-h/r}$

Where:
- C(h) = covariance
- h = separation distance
- r = range

Spatial Statistics
Spatial Covariance Models

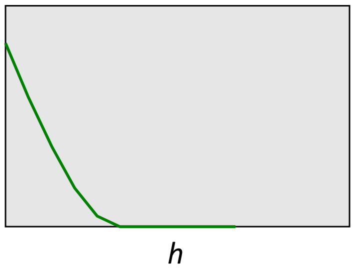


Gaussian Model:

$C(h) = [\sigma^2 \exp(-h^2/r^2)]$

Where:
- C(h) = covariance
- h = separation distance
- r = range

Spatial Statistics
Spatial Covariance Models

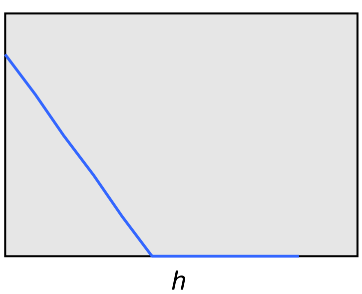


Spherical Model:

$C(h) = \sigma^2[1 - (3h/2r) + h^3/2r^3)]$, if $h < r$

$C(h) = 0$, if $h > r$

Spatial Statistics
Spatial Covariance Models



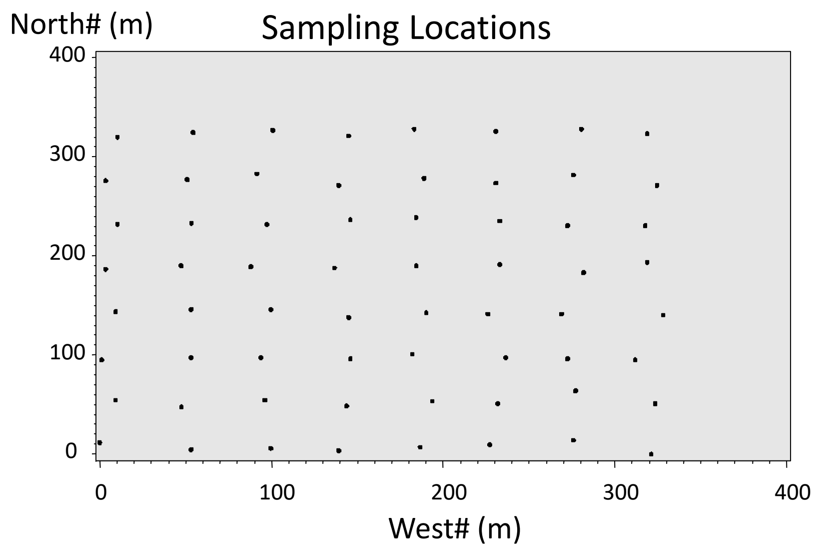
Linear Model:

$C(h) = \sigma^2[1 - hr]$, if $h < 2/r$

$C(h) = 0$, if $h > 2/r$

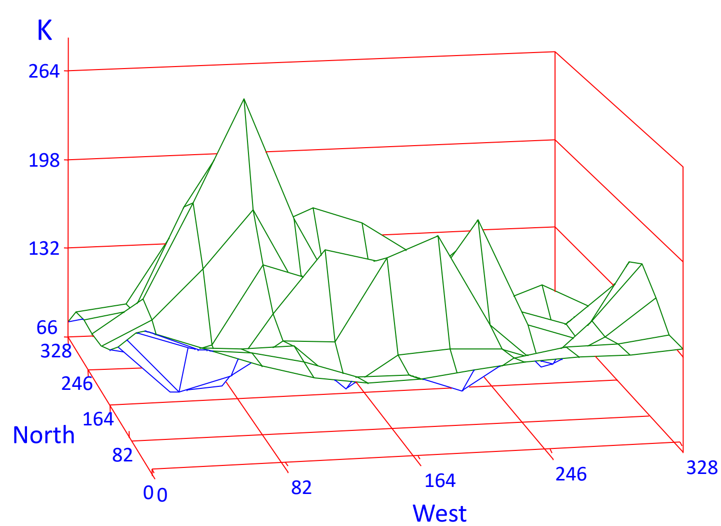
Spatial Statistics

Example: Soil K



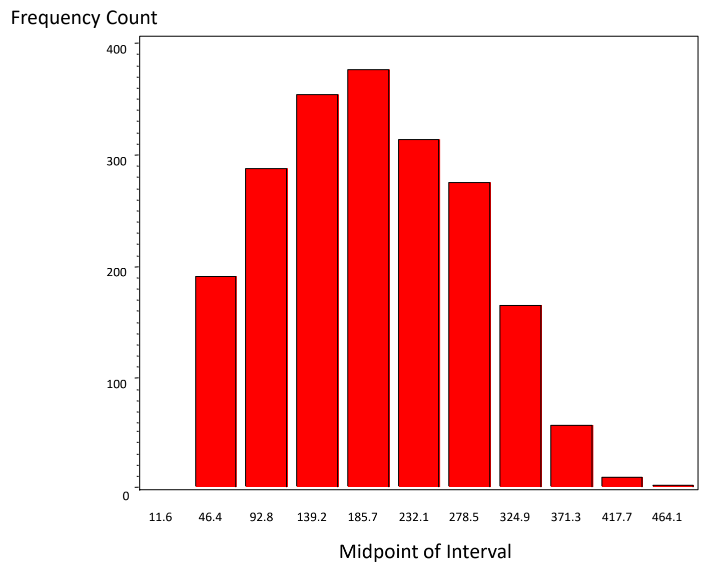
Spatial Statistics

Example: Spatial Distribution



Spatial Statistics

Example: Distribution of Pairwise Distances



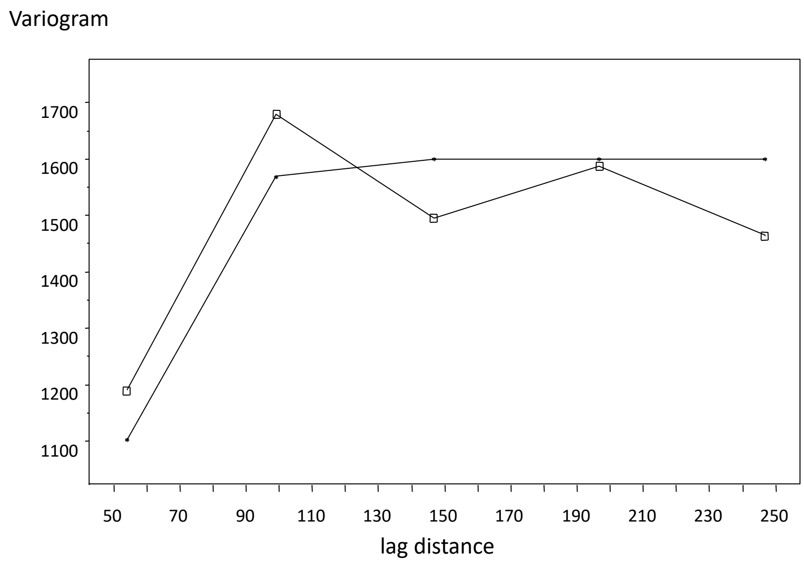
Spatial Statistics

Example: SAS VARIOGRAM Procedure

```
proc variogram data=a outvar=b;
  compute lagdistance=50 maxlag=5;
  coordinates xc=west yc=north;
  var k;
run;
```

Spatial Statistics

Example: Sample Semivariogram



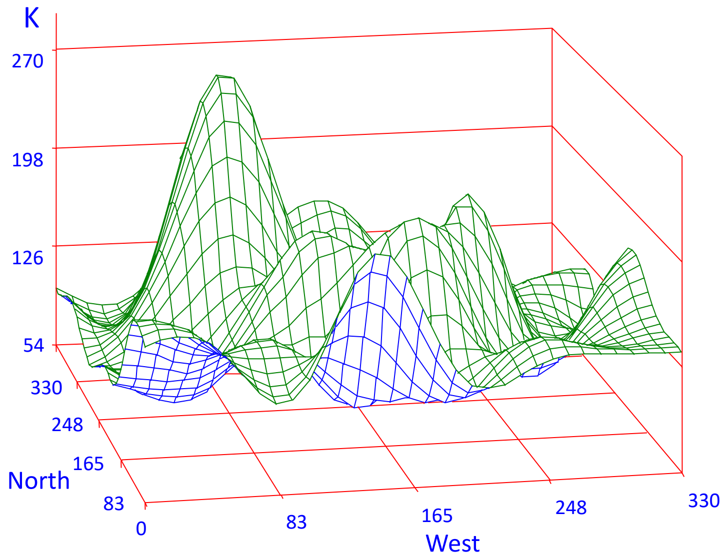
Spatial Statistics

Example: SAS KRIGE2D Procedure

```
proc krige2d data=a;
  pred var=k r=60;
  model scale=1600 range=50 form=gauss;
  coord xc=west yc=north;
  grid x=0 to 330 by 10 y=0 to 330 by 10;
run;
```

Spatial Statistics

Example: Kriged K Concentrations



Spatial Correlation

Standard Error of a Mean Difference

$$S_{\bar{d}} = \sqrt{(S^2 - Cov)\left[\frac{1}{n_1} + \frac{1}{n_2}\right]}$$

Spatial Statistics

Example: Wheat Variety Trial

Littell et al., 1996, p. 321ff.

RCBD Model:

$Y_{ijk} = \mu + B_i + T_j + \varepsilon_{ij}$

Assuming independence

Alternative Model:

$Y_{ijk} = \mu + T_j + \varepsilon_{ij}$

Assuming spatial correlation

Spatial Statistics

Example: Wheat Variety Trial

RCBD Analysis:

```
proc mixed data=wheat;
   class rep name;
   model y=name;
   random rep;
run;
```

Spatial Analysis:

```
proc mixed data=wheat noprofile;
   class name;
   model y=name;
   parms (61.6) (18.1) / noiter;
   repeated / subject=intercept type=sp(sph)(lat lng);
run;
```

Example from Littell et al., 1996, p. 321ff.

Spatial Statistics

Example: Wheat Variety Trial

RCBD Analysis:

```
Type 3 Tests of Fixed Effects

              Num     Den
Effect         DF      DF    F Value    Pr > F

name           55     165       0.88    0.7119

Estimates
                            Standard
Label        Estimate     Error      DF    t Value    Pr > |t|

arap v brul   3.3625     4.9791     165     0.68       0.5004
arap v buck   3.8750     4.9791     165     0.78       0.4375
arap v ks83   5.3125     4.9791     165     1.07       0.2875
brul v ks83   1.9500     4.9791     165     0.39       0.6958
```

Example from Littell et al., 1996, p. 321ff.

Spatial Statistics

Example: Wheat Variety Trial

Spatial Analysis:

```
Type 3 Tests of Fixed Effects

              Num     Den
Effect         DF      DF    F Value    Pr > F

name           55     168       6.09    0.0001

Estimates
                            Standard
Label         Estimate    Error      DF    t Value    Pr > |t|

arap v brul   1.6278     1.9005     168     0.86       0.3929
arap v buck  -9.1324     1.9748     168    -4.62       0.0001
arap v ks83   0.4385     1.9233     168     0.23       0.8199
brul v ks83  -1.1894     1.9197     168    -0.62       0.5364
```

Example from Littell et al., 1996, p. 321ff.

Spatially Balanced Designs

Average Distance

Randomized Complete Block Design

Blocks 1, 2, 3, 4

Treatments A, B, C, D

Block	**Treatment**				
1	C	A	B	D	d = 1
2	B	D	C	A	d = 2
3	A	B	D	C	d = 3
4	D	B	C	A	

Example from Gbur and Christman, Spatial statistics in agricultural research workshop. CSSA Annual Meeting, 2006.

Spatially Balanced Designs

Average Distance

Distance between plots

Contrast	Block				Mean
	1	2	3	4	
A vs B	1	3	1	2	1.75
A vs C	1	1	3	1	1.50
A vs D	2	2	2	3	2.25
B vs C	2	2	2	1	1.75
B vs D	1	1	1	1	1.00
C vs D	3	1	1	2	1.75
Mean					1.67
CV					24.50

Spatially Balanced Designs
Average Distance

Block	Treatment			
1	C	A	B	D
2	B	D	C	A
3	A	B	D	C
4	D	B → A	C	A → B

Spatially Balanced Designs
Average Distance

Distance between plots

Contrast	Block				Mean
	1	2	3	4	
A vs B	1	3	1	2	1.75
A vs C	1	1	3	1	1.50
A vs D	2	2	2	1	1.75
B vs C	2	2	2	1	1.75
B vs D	1	1	1	3	1.50
C vs D	3	1	1	2	1.75
Mean					1.67
CV					7.75

Spatially Balanced Designs
Nearest Neighbor Approach

Block	Treatment			
1	C	A	B	D
2	B	D	C	A
3	A	B	D	C
4	D	B	C	A

Spatially Balanced Designs
Nearest Neighbor Approach

Neighbors and Distance between plots

Contrast	Neighbors	Block				Mean
		1	2	3	4	
A vs B	2	1	3	1	2	1.75
A vs C	3	1	1	3	1	1.50
A vs D	0	2	2	2	3	2.25
B vs C	1	2	2	2	1	1.75
B vs D	4	1	1	1	1	1.00
C vs D	2	3	1	1	2	1.75
Mean						1.67
CV						24.50

Spatially Balanced Designs

Nearest Neighbor Approach

Block	Treatment			
1	C C	A A	B B	D D
2	B B	D D	C A	A C
3	A A	B D	D C	C B
4	D D	B C	C B	A A

- 12 neighbor pairs: $r(t - 1)$
- 6 treatment pairs: $t(t - 1)/2$

Spatially Balanced Designs

Spatially-Balanced Complete Block Design (SBCB)

Neighbors and Distance between plots

Contrast	Neighbors	Block 1	Block 2	Block 3	Block 4	Mean
A vs B	2	1	2	3	1	1.75
A vs C	2	1	1	2	2	1.50
A vs D	2	2	1	1	3	1.75
B vs C	2	2	3	1	1	1.75
B vs D	2	1	1	2	2	1.50
C vs D	2	3	2	1	1	1.75
Mean						1.67
CV						7.75